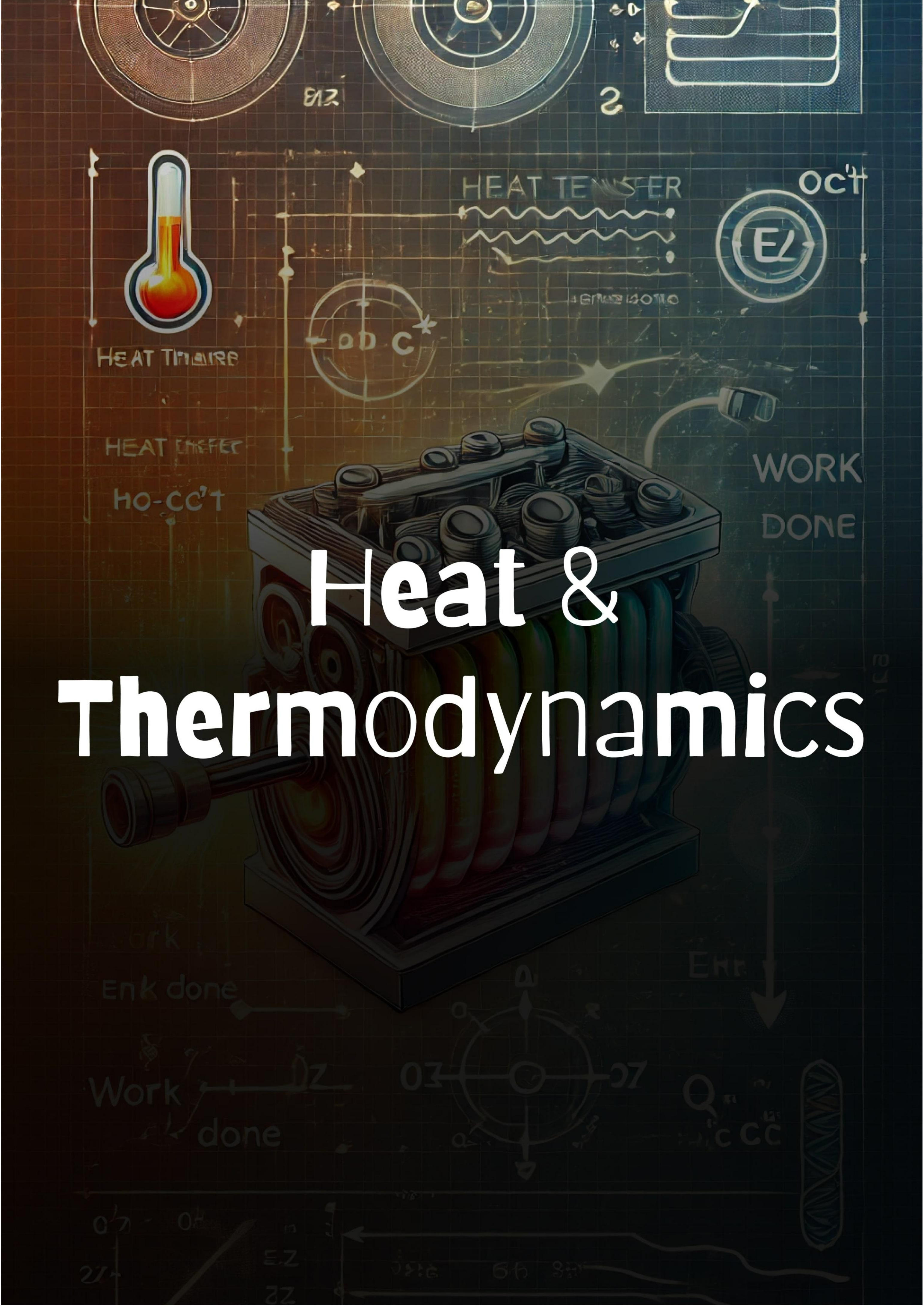


Heat & Thermodynamics

Heat & Thermodynamics

THERMAL EXPANSION

Whenever a typical isotropic substance is uniformly heated, the vibration of the molecules/atoms increases. This increased vibration leads to an increase in the inter-molecular separation, thus the material expands isotropically.

NOTE: Thermal expression is like photographic enlargement in space i.e. each linear dimension gets multiplied by the same factor.

COEFFICIENT OF LINEAR THERMAL EXPANSION (α)

Consider a rod of original length L which expands by an amount ΔL upon heating through a temperature (ΔT). Then we define the coefficient of linear thermal expression as

$$\boxed{\alpha = \frac{\Delta L}{L \Delta T}}$$

It can also be thought as linear thermal strain per unit increase in temperature.

$$\alpha = \frac{\epsilon_T}{\Delta T}$$

The more useful form is

$$L' = L(1+\alpha\Delta T)$$

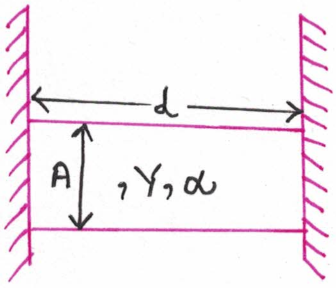
Que.) Rod of length 'L', cross-section 'A' & Young's Modulus/ just slips b/w two rigid walls. If the rod is now heated through a temp. ΔT. Find the force with which the rod pushes the wall a part.

Natural length : $L(1+\alpha\Delta T)$

Actual length $= L$

Compression $= L\alpha\Delta T$

$$\epsilon_c = \frac{L\alpha\Delta T}{L} = \alpha\Delta T$$

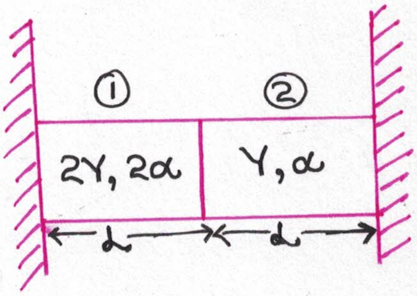


$$\sigma_c = Y\epsilon_c$$

$$\sigma_c = Y\alpha\Delta T = \frac{F_c}{A}$$

$$F_c = AY\alpha\Delta T$$

Que.) A compound rod just slips b/w two rigid walls as shown in the figure, Find:
(i) The shift in the interface
(ii) Compressive stress in the rod.



① $N.L. = l(1 + 2\alpha\Delta T)$

$A.L. = l + x$

$$\epsilon_1 = \frac{l \times 2\alpha\Delta T - x}{l}$$

$$\sigma_1 = 2Y\left(\frac{2l\alpha\Delta T - x}{l}\right)$$

Similarly, $\sigma_2 = Y\left(\frac{l\alpha\Delta T + x}{l}\right)$

as area of cross section is same and force b/w the molecules will also be same.

$$\sigma_1 = \sigma_2$$

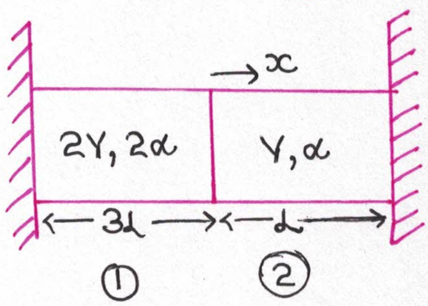
$$\frac{2\cancel{Y}}{\cancel{l}}(2l\alpha\Delta T - x) = \frac{\cancel{Y}}{\cancel{l}}(l\alpha\Delta T + x)$$

$$3x = 3l\alpha\Delta T$$

(i) $x = l\alpha\Delta T$

(ii) $\sigma = 2Y\alpha\Delta T$

Que.) Repeat the previous problem, for the following data:



$$\epsilon_1 = \frac{3l\alpha\Delta T - x}{3l}$$

$$\epsilon_2 = \frac{l\alpha\Delta T + x}{l}$$

$$\sigma_1 = \sigma_2$$

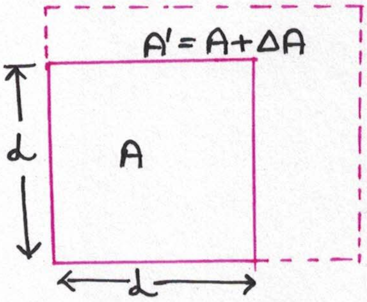
$$2\cancel{Y}\left(\frac{6l\alpha\Delta T - x}{3\cancel{l}}\right) = \cancel{Y}\left(\frac{l\alpha\Delta T + x}{\cancel{l}}\right)$$

$$5x = 9 l \alpha \Delta T$$

$$x = \frac{9}{5} l \alpha \Delta T$$

COEFFICIENT OF AREAL THERMAL EXPANSION (β)

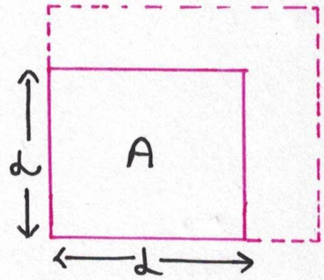
Areal strain per unit increase in temperature is called Coefficient of aerial thermal expansion A more useful form of aerial strain is



$$A' = A(1+\beta \Delta T)$$

$$\frac{\Delta A}{A} \times \frac{1}{\Delta T} = \beta$$

Que.) Show that for a typical material $\beta = 2\alpha$



$$A' = A(1+\beta \Delta T)$$

$$A' = l'^2$$

$$A' = l^2(1+\alpha \Delta T)^2$$

$$A' = A(1+2\alpha \Delta T)$$

$$\therefore \beta = 2\alpha$$

Any lamina can be considered as pieces of squares joined together.

Que.) A presmatic vessel has a base area of 'A' & volume 'v'. What will be the final volume of the vessel if the material has a Coefficient of linear expansion 'α' in terms of heat change.

$$A' = A(1+2\alpha \Delta T)$$

$$H' = H(1+\alpha \Delta T)$$

$$A'H' = AH(1+3\alpha \Delta T)$$

$$v' = v(1+3\alpha \Delta T)$$

COEFFICIENT OF VOLUMETRIC THERMAL EXPANSION (γ)

Volumetric strain per unit increase in temperature is called coefficient of volumetric thermal expansion.

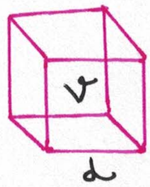
Mathematically,

$$\gamma = \frac{\Delta v}{v} \times \frac{1}{\Delta T}$$

More useful form of volumetric strain is

$$\gamma' = v(1+\gamma\Delta T)$$

Que.) Show that for a typical material $\gamma = 3\alpha$



$$v' = v(1+\gamma\Delta T)$$

$$v' = l^3(1+\alpha\Delta T)^3$$

$$v' = v(1+3\alpha\Delta T)$$

$$\gamma = 3\alpha$$

NOTE: whenever a vessel of capacity 'v' is heated, the new capacity becomes

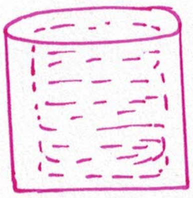
$$v' = v(1+\gamma_m \Delta T)$$

γ_m is the γ of the material of the vessel.

Que.) A vessel of volume 'v' is filled to the brim with a liquid as shown, find relation b/w γ_m and γ_L

(i) If no liquid is to spill on heating.

(ii) If no liquid is to spill on cooling.



$$v'_{vessel} = v(1+\gamma_m \Delta T)$$

$$v'_{liquid} = v(1+\gamma_L \Delta T)$$

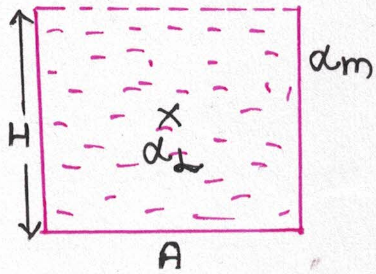
(i) $v'_{vessel} \geq v'_{liquid}$

$$\gamma_m \geq \gamma_{liquid}$$

(ii) Similarly,

$$\gamma_m \leq \gamma_{liquid}$$

Que.) A vessel of height 'H' and cross-section area 'A' is filled with a liquid to the top, find the relation b/w α_m and α_l so that the height of liquid does change with temperature.

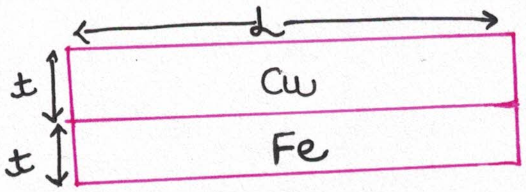


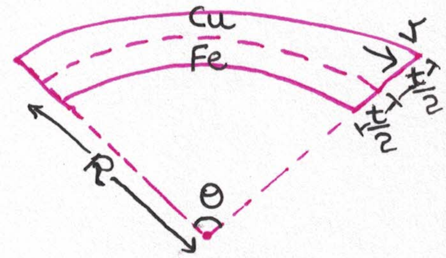
$$v'_l = A(1+2\alpha_m \Delta T)H$$

$$= AH(1+3\alpha_L \Delta T)$$

$$\alpha_m = \frac{3}{2}\alpha_L$$

Que:) Find radius of curvature of bi-mettalic strip, when the temp. is increased by ΔT.





$$l'_{cu} = l(1+\alpha_{cu}\Delta T) = \left(R+\frac{t}{2}\right)\theta$$

$$l'_{Fe} = l(1+\alpha_{Fe}\Delta T) = \left(R-\frac{t}{2}\right)\theta$$

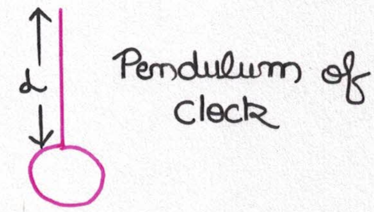
$$\frac{1+\alpha_{cm}\Delta T}{1+\alpha_{Fe}\Delta T} = \frac{(R+t/2)}{(R-t/2)}$$

$$\frac{N-D}{N} = \frac{(\alpha_{cu}-\alpha_{Fe})\Delta T}{1+\alpha_{Fe}\Delta T} = \frac{t}{(R-t/2)}$$

$$\left(R-\frac{t}{2}\right) = \frac{(1+\alpha_{Fe}\Delta T)t}{(\alpha_{cm}-\alpha_{Fe})\Delta T}$$

$$R \approx \frac{t}{(\alpha_{cm}-\alpha_{Fe})\Delta T}$$ ($t/2$ is small as compared to R)

Que:) How much time will the clock loose if temperature is increased by $1^\circ C$.



$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T' = 2\pi\sqrt{\frac{l(1+\alpha\Delta T)}{g}}$$

Seconds loose in a day $= \dfrac{D}{T} - \dfrac{D}{T'}$

$$= D\left[\frac{T'-T}{TT'}\right]$$

$$T = 2\pi\sqrt{l/g}$$

$$\ln T = \frac{1}{2}\ln l - \frac{1}{2}\ln g + \ln 2\pi$$

$$\frac{dT}{T} = \frac{1}{2}\cdot\frac{dl}{l}$$ (differentiate)

$$\therefore \frac{\Delta T}{T} \approx \frac{1}{2}\frac{\Delta l}{l} \approx \frac{\alpha \Delta \theta}{2} \qquad (\Delta\theta \rightarrow \text{Change in temp})$$

$$\Delta T = \frac{T\alpha\Delta\theta}{2}$$

$$\therefore \text{Seconds loose} = D\left[\frac{\cancel{T}\alpha\Delta\theta}{2\cancel{T}T'}\right]$$

$$= D\left(\frac{\alpha\Delta\theta}{2T'}\right)$$

HEAT TRANSFER

There are three modes of heat transfer:

CONDUCTION

When the energy is transmitted from one part of substance to another part, due to thermal oscillations of the molecule without bulk transfer of material, we call it conduction.

Eg: When one end of rod is heated, other end also becomes hot.

CONVECTION

When the energy is transmitted from one point in space to another point due to physical motion of energetic particles of the bulk of the material, we call it Convection.

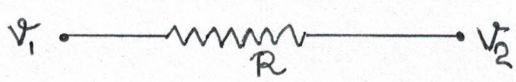
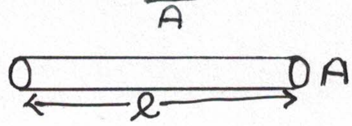
Eg: Boiling of water.

RADIATION

When the energy is transmitted from one point in space to another by virtue of electromagnetic radiation, we call it radiative heat transfer.

Eg: Earth receiving energy from Sun.

ANALOGY B/W ELECTRIC CURRENT & THERMAL CURRENT

ELECTRIC CURRENT	THERMAL CURRENT
	
$\frac{dV}{dt} = I = \frac{V_1 - V_2}{R}$	$\frac{d\theta}{dt} = I_T = \frac{T_1 - T_2}{R_T}$
$R = \frac{\rho l}{A}$	$R_T = \frac{l}{KA}$
	$K \rightarrow$ thermal Conductivity

SERIES CONNECTION

Two Conductors are said to be connected in series if by virtue of configuration they carry same current.

$$R_{eff.} = R_1 + R_2 + \text{------}$$

PARALLEL CONNECTION

Two conductors are said to be connected in parallel if by virtue of configuration, they have the same temperature difference (potential difference).

$$R_{eff.} = \frac{1}{R_1} + \frac{1}{R_2} + \text{--------}$$

Que.) What is the unit of Thermal Resistance.

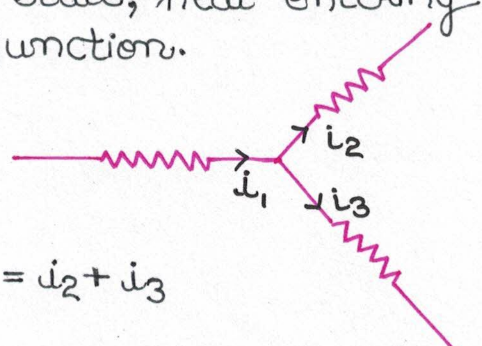
$$\frac{dQ}{dt} = \frac{T_1 - T_2}{R_T} \qquad Q \rightarrow \text{heat}$$

$$R_T = K\text{-sec}/J$$

KIRCHOFF'S LAWS

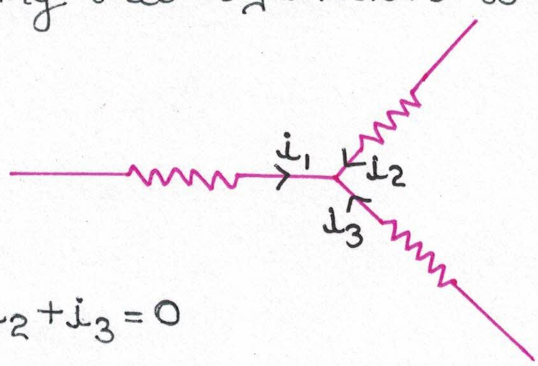
1. KIRCHOFF'S JUNCTION LAW

In steady state, heat entering a junction is equal to heat leaving a junction.



$$i_1 = i_2 + i_3$$

Net heat flowing into a junction is 0.

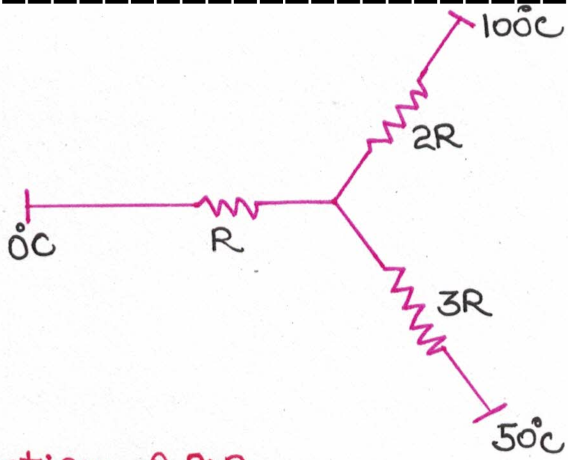


$$i_1 + i_2 + i_3 = 0$$

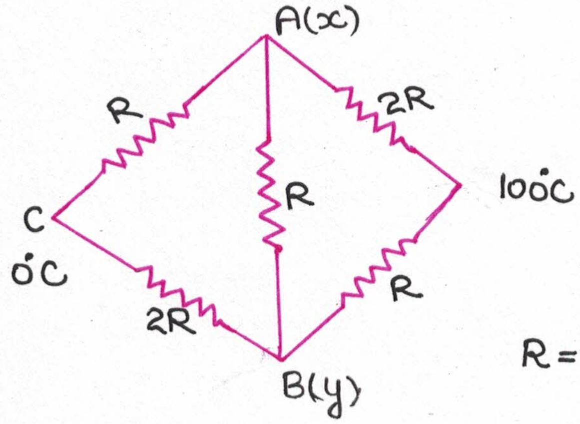
Que.) Calculate the temperature of the junction.

$$\frac{100 - x}{2R} + \frac{0 - x}{R} + \frac{50 - x}{3R} = 0$$

$$3x - 300 + 2x - 100 + 6x = 0$$
$$11x - 400 = 0$$
$$x = \frac{400}{11}°C$$



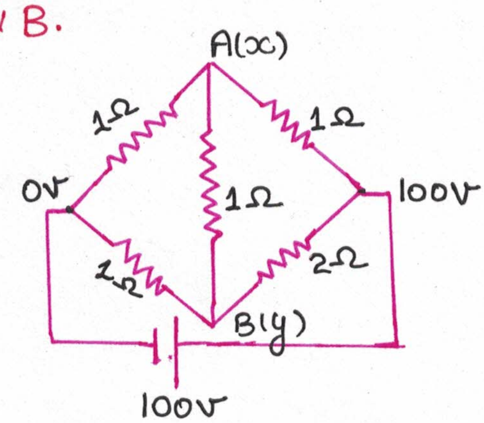
Que.) Calculate the temperature of junction A & B.



R = 1

$$\frac{x-0}{R} + \frac{x-y}{R} + \frac{x-100}{2R} = 0$$
$$2x + 2x - 2y + x - 100 = 0$$
$$5x - 2y - 100 = 0$$
$$\frac{y-0}{2R} + \frac{y-x}{R} + \frac{y-100}{R} = 0$$
$$y + 2y - 2x + 2y - 200 = 0$$
$$5y - 2x - 200 = 0$$
$$25y - 10x - 1000 = 0$$
$$21y = 1200$$
$$y = \frac{1200}{21} = \frac{400}{7}°C$$
$$x = \frac{300}{7}°C$$

Que.) Calculate x & y, which are at A & B.



$$x + x - y + x - 100 = 0$$
$$3x - y - 100 = 0 \quad —(1)$$
$$y + y - x + \frac{y-100}{2} = 0$$
$$5y - 2x - 100 = 0 \quad —(2)$$
$$5 \times (1) \Rightarrow 15x - 5y - 500 = 0$$
$$+(2) \Rightarrow 5y - 2x - 100 = 0$$
$$13x = 600$$
$$x = \frac{600}{13} V, \quad y = \frac{500}{13} V$$

2. KIRCHOFF'S LOOP LAW (VOLTAGE LAW)

Statement: Summation of Potential drop along any close loop.

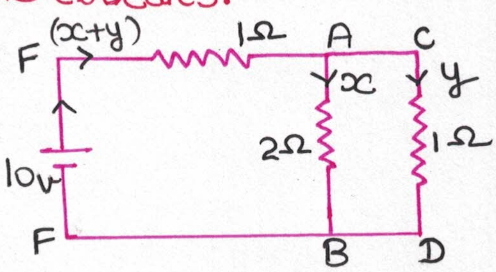
NOTE: Potential drop can be taken as same as temperature drop for thermal connections.

To apply loop law:

(i) If you hit the positive terminal of battery, write $+V$ and if we hit negative terminal then we can write $-V$ where V is the emf of the battery.

(ii) If we hit the register in direction of current, we write '$+IR$' and '$-IR$' if we hit in opposite direction.

Que) Find current flowing in the circuits.



ABCD loop

$$2x - y = 0$$

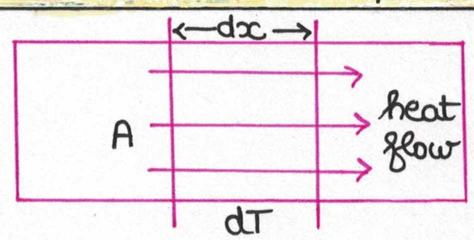
FABEF loop

$$3x + y = 10$$

$$x = 2 \;\&\; y = 4$$

DIFFERENTIAL FORM OF HEAT CONDUCTION EQUATION

$$\boxed{\frac{dQ}{dt} = -KA\frac{dT}{dx}}$$

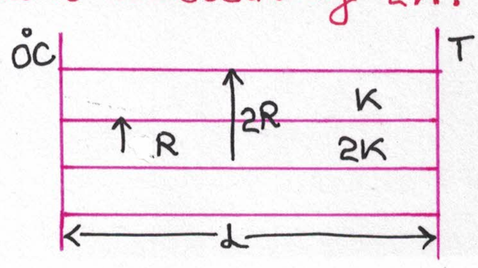


(moving in the direction of heat flow, temperature decreased thus we have negative sign in the eqn.)

Que) A cylinderical shell of radius 2R and conductivity K is filled with the core of radius R and conductivity 2K. Find heat flow

(i) Through the core

(iii) Through the shell



$$R_{Shell} = \frac{d}{K \times 3\pi r^2}$$

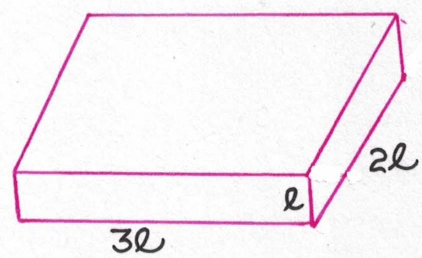
$$R_{Core} = \frac{d}{2K\pi R^2}$$

$$\frac{d\theta}{dt}_{shell} = \frac{T}{R_{shell}} = \frac{3\pi R^2 KT}{d}$$

For Core, $\frac{d\theta}{dt} = \frac{2K\pi R^2 T}{d}$

$$\therefore \frac{dQ}{dt}_{Total} = \frac{5\pi R^2 KT}{d}$$

Que.) Find ratio of Max. to min. resistance of cube.

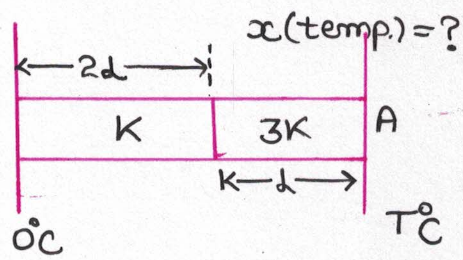


$$\text{Max } R = \frac{3l}{KA} = \frac{3l}{2Kl^2}$$

$$\text{Min } R = \frac{l}{KA} = \frac{l}{K6l^2}$$

Ratio = 9:1

Que.) Calculate x and total heat transfer for the shown situation.



$$\frac{x \times KA}{2d} + \frac{(x-T)3KA}{d} = 0$$

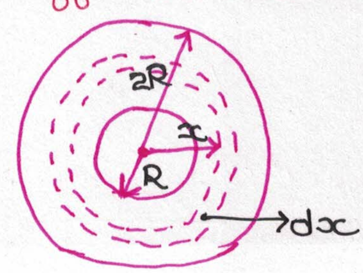
$$T x KA - 6TKA = 0$$

$$x = \frac{6T}{7}$$

Total heat transfered $= \dfrac{6T/7 \times KA}{2d}$

$$= \frac{3TKA}{7d}$$

Que.) For the shown Spherical shell, find the effective resistance



$$dR' = \frac{dx}{4\pi x^2}$$

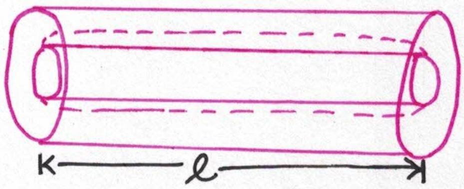
$$\int_0^{R_{eff.}} dR' = \int_R^{2R} \frac{dx}{x^2} \times \frac{1}{K4\pi}$$

$$R' = \frac{1}{4\pi K}\left(\frac{1}{R} - \frac{1}{2R}\right)$$

$$R' = \frac{1}{8\pi R K}$$

Que.) Find effective resistance of cylindrical shell.

$$dR' = \frac{1}{K 2\pi x l} \cdot dx$$



$$\int_0^{R_{eff}} dR' = \int_R^{2R} \frac{1}{2\pi K l} \times \frac{1}{x} \cdot dx$$

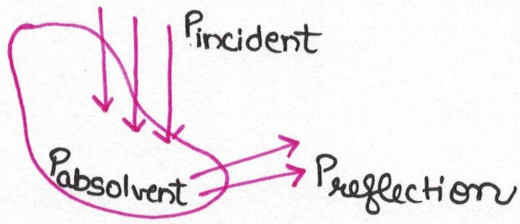
$$R_{eff.} = \frac{1}{2\pi K l} \ln\left(\frac{2\cancel{R}}{\cancel{R}}\right)$$

$$R_{eff} = \frac{\ln 2}{2\pi K l}$$

RADIATION

ABSORPTIVITY

The ratio of absorbed power to the incident power is called absorptivity. (absorptive power)



$$a = \frac{P_{abs}}{P_{inc.}}$$

NOTE: Absorptivity is the surface property.

EMISSIVITY

The ratio of emitted power to the incident power when a body is in thermal equilibrium under the radiative heat transfer only is called emissivity (or emissive power).

$$E = \left.\frac{P_{emitted}}{P_{incident}}\right|_{equilibrium}$$

NOTE: For thermal equilibrium, absorped power must be equal to emitted power
∴ For any object $E = a$

BLACK BODY

A body which absorbs 100% of the incident radiation is called a black body.

For black body, $\epsilon = a = 1$

DERIVATION OF NEWTON'S LAW OF COOLING

$$\frac{dQ}{dt} = \epsilon \sigma A (T^4 - T_0)^4$$

If $T = T_0 + \Delta T$

$$\frac{dQ}{dt} = \epsilon \sigma A \left\{ (T_0 + \Delta T)^4 - T_0^4 \right\}$$

$$\frac{dQ}{dt} = \epsilon \sigma A \left\{ T_0^4 \left(1 + \frac{\Delta T}{T_0}\right)^4 - T_0^4 \right\}$$

If $\Delta T << T_0$, then

$$\frac{dQ}{dt} = 4 \epsilon \sigma A T_0^3 \Delta T$$

$$\frac{dQ}{dt} = 4 \epsilon \sigma A T_0^3 (T - T_0) \quad \text{— (1)}$$

also, $$\frac{dQ}{dt} = -mc\frac{dT}{dt} \quad \text{— (2)}$$

using (1) & (2)

$$mc\frac{dT}{dt} = -4 \epsilon \sigma A T_0^3 (T - T_0)$$

$$\frac{dT}{dt} = \frac{-4 \epsilon \sigma A T_0^3}{mc} (T - T_0)$$ (C is molar heat capacity)

$$\frac{dT}{t} = -K(T - T_0)$$ $\left(K = \frac{4 \epsilon \sigma A T_0^3}{mc}\right)$

STEFAN'S LAW (For black body radiation)

Power emitted by a black body per unit surface area is given by

$$\boxed{\frac{P_{emi}}{A} = \sigma T^4} = I \text{ (intensity)}$$

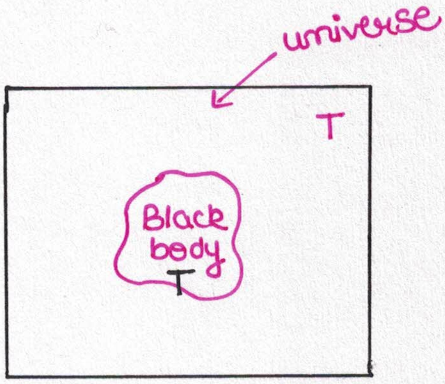
where σ is Stefan's constant and T is temperature of the body.

$$\sigma = 5.67 \times 10^{-3} \ W/m^2K^4$$

$P_{emit} = \sigma A T^4$ (for black body)

$P_{inc} = \sigma A T^4$ (for black body)

$P_{abs} = \sigma A T^4$ (for black body)



Now if temp. of body is increased to T_H then

$P_{emit} = \sigma A T_H^4$

$P_{abs} = \sigma A T^4$

Net rate of heat emitted is

$P_{net} = P_{emit} - P_{abs}$

$P_{net} = \sigma A (T_H^4 - T^4)$

Now If we replace black body by grey body (general body) then

$P_{inc} = \sigma A T^4$

$P_{abs} = P_{emit} = \epsilon \sigma A T^4$

Temp. increased to T_H

$P_{emit} = \epsilon \sigma A T_H^4$

$P_{abs} = \epsilon \sigma A T^4$

$P_{net} = P_{emit} - P_{abs}$

$\boxed{P_{net} = \epsilon \sigma A (T_H^4 - T^4)}$

Que.) A thin plate of side 10cm is heated to a temp. of 800°C. If $\epsilon = 0.6$, what is rate of emission?

$P_{inc} = \sigma A T^4$

$P_{emit} = \epsilon \sigma A T^4$

$= 5.67 \times 10^{-8} \times \frac{1}{10} \times \frac{1}{10} \times 0.6 \times (.1073)^4$

Que.) Show that for small temp. differences b/w surrounding & body, the net radiated power is linearly proportional to ΔT where the ΔT is temp. difference b/w body & surroundings.

$P_{net\ out} = \epsilon \sigma A \left((T_0 + \Delta T)^4 - T_0^4 \right)$

$= \epsilon \sigma A T^4 \left(\left(1 + \frac{\Delta T}{T}\right)^4 - 1 \right)$

As ΔT is small, using binomial expansion

$$\boxed{P_{net\ out} = \epsilon \sigma A T^3 \Delta T}$$

WEIN'S DISPLACEMENT LAW

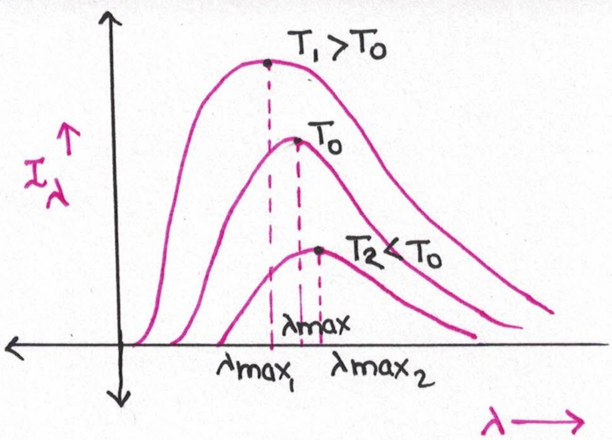
SPECTRAL INTENSITY

Power received from unit area per unit wave length around an infinitesimal interval of wavelength at a given wavelength is called spectral intensity. Mathematically, if dI is the intensity b/w wavelength λ and $(\lambda + d\lambda)$, then spectral intensity at that λ is,

$$I_\lambda = \frac{dI}{d\lambda}$$

It is obvious that area under the spectral intensity vs wavelength graph gives total intensity.

SPECTRAL INTENSITY Vs WAVELENGTH GRAPH



λ_{max} is the wavelength for which spectral intensity is max

WEIN'S DISPLACEMENT LAW

$$\lambda_{max} \times T = b$$

where b is called weins constant

$$(b = 2.9\,mm\,K)$$

Que.) Surface of Sun has a temp. of 5800K. For what wavelength would you obtain max. spectral intensity. (Now you know why sunlight appears yellow)

$$\lambda_{max} = \frac{2.9}{5800} = 5 \times 10^{-4}\,mm = 500\,nm$$

∴ yellow color has more fraction.

also, our eyes perceive yellow color, more easily.

NEWTON'S LAW OF COOLING

Rate of cooling of a body near room temp. is linearly proportional to the temp. difference of the body from the room temp.

Mathematically,

$$\boxed{\frac{dT}{dt} = -K(T-T_0)}$$

where T is the instantaneous temp. of the body & T_0 is ambient (surrounding) temperature, t is time and K is the proportionality constant and depends on geometry of body in relation to surroundings.

APPROXIMATE FORM OF NEWTON'S LAW OF COOLING

$$\boxed{\frac{\Delta T}{\Delta t} = -K(T_{mean}-T_0)}$$

Here, ΔT is the total temp. change over the cooling range, T_{mean} is the average temp. of the body during the cooling range.

$$T_{mean} = \frac{T_i+T_f}{2}$$

Que.) Temperature of a body falls from 40°c to 36°c in 5min. Find the additional time required to cool it to 32°c. (Given ambient temp. is 16°c)

$$\frac{4}{5} = -K(38-16) \quad \text{—①}$$

$$\frac{4}{\Delta t} = -K(34-16) \quad \text{—②}$$

$$\frac{\Delta t}{5} = \frac{\cancel{22}\,^{11}}{\cancel{18}\,_{9}} \qquad \text{(dividing ① by ②)}$$

$$\Delta t = \frac{55}{9} = 6.1\text{ min}$$

by exact form of Newton's law,

$$\int_{T_i}^{T_f} \frac{dT}{T-T_0} = \int_0^t -K.\,dt$$

$$\ln\left(\frac{T_f - T_0}{T_i - T_0}\right) = -Kt$$

$$\ln\left(\frac{36-16}{40-16}\right) = -K \times x_{min}$$

$$\frac{x}{5} = \frac{\ln(16/20)}{\ln(20/24)}$$

$$= \frac{\ln(4/5)}{\ln(5/6)}$$